8.1 Volumes of Cylinders

Essential Question How can you find the volume of a cylinder?

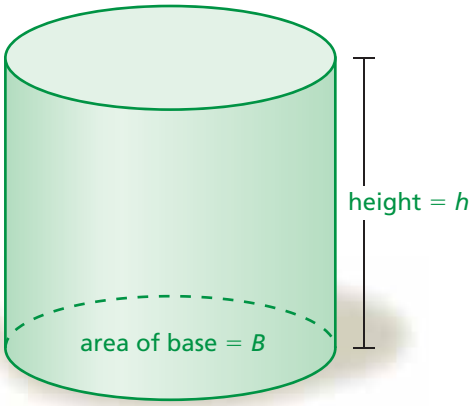
1 ACTIVITY: Finding a Formula Experimentally

Work with a partner.

a. Find the area of the face of a coin.

b. Find the volume of a stack of a dozen coins.

c. Write a formula for the volume of a cylinder.



2 ACTIVITY: Making a Business Plan

Work with a partner. You are planning to make and sell three different sizes of cylindrical candles. You buy 1 cubic foot of candle wax for $20 to make 8 candles of each size.

a. Design the candles. What are the dimensions of each size of candle?

b. You want to make a profit of $100. Decide on a price for each size of candle.

c. Did you set the prices so that they are proportional to the volume of each size of candle? Why or why not?

Common Core

Geometry

In this lesson, you will

- find the volumes of cylinders.
- find the heights of cylinders given the volumes.
- solve real-life problems.

Learning Standard 8.G.9

3 ACTIVITY: Science Experiment

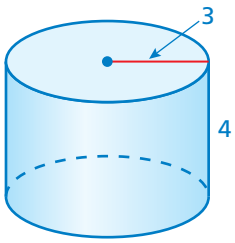
Work with a partner. Use the diagram to describe how you can find the volume of a small object.

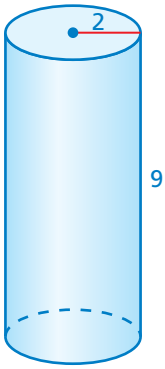
4 ACTIVITY: Comparing Cylinders

Work with a partner.

a. Just by looking at the two cylinders, which one do you think has the greater volume? Explain your reasoning.

b. Find the volume of each cylinder. Was your prediction in part (a) correct? Explain your reasoning.





Math Practice 1

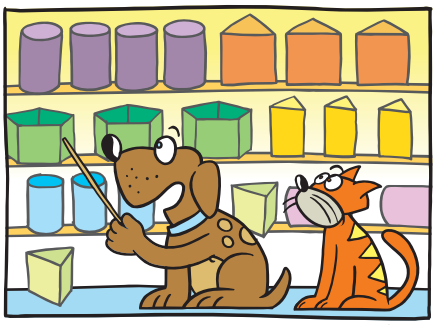
Consider Similar Problems

How can you use the results of Activity 1 to find the volumes of the cylinders?

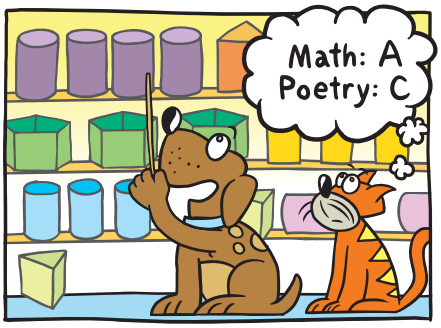
What Is Your Answer?

5. IN YOUR OWN WORDS How can you find the volume of a cylinder?

6. Compare your formula for the volume of a cylinder with the formula for the volume of a prism. How are they the same?



"Here's how I remember how to find the volume of any prism or cylinder."



"Base times tall, will fill 'em all."

Practice Use what you learned about the volumes of cylinders to complete Exercises 3–5 on page 338.

8.1 Lesson

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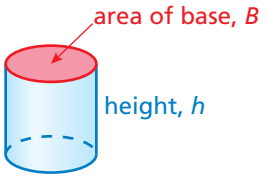
Key Idea

Volume of a Cylinder

Words The volume V of a cylinder is the product of the area of the base and the height of the cylinder.



Algebra $V = Bh$

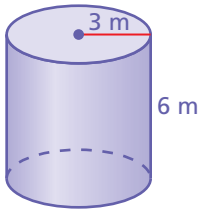
Area of base — Height of cylinder

EXAMPLE 1 Finding the Volume of a Cylinder

Find the volume of the cylinder. Round your answer to the nearest tenth.

$V = Bh$	Write formula for volume.
$= \pi(3)^2(6)$	Substitute.
$= 54\pi \approx 169.6$	Use a calculator.

∴ The volume is about 169.6 cubic meters.



Study Tip

Because $B = \pi r^2$, you can use $V = \pi r^2 h$ to find the volume of a cylinder.

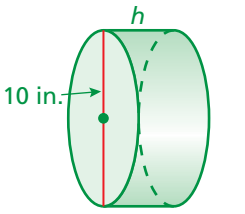
EXAMPLE 2 Finding the Height of a Cylinder

Find the height of the cylinder. Round your answer to the nearest whole number.

The diameter is 10 inches. So, the radius is 5 inches.

$V = Bh$	Write formula for volume.
$314 = \pi(5)^2(h)$	Substitute.
$314 = 25\pi h$	Simplify.
$4 \approx h$	Divide each side by 25π.

∴ The height is about 4 inches.



On Your Own

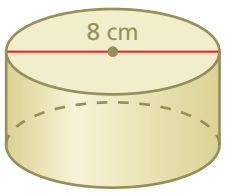
Now You're Ready
Exercises 3–11 and 13–15

Find the volume V or height h of the cylinder. Round your answer to the nearest tenth.

1.

$V \approx$ ____

2.



$h \approx$ ____

EXAMPLE 3 Real-Life Application

How much salsa is missing from the jar?

The empty space in the jar is a cylinder with a height of $10 - 4 = 6$ centimeters and a radius of 5 centimeters.

$V = Bh$ Write formula for volume.

$= \pi(5)^2(6)$ Substitute.

$= 150\pi \approx 471$ Use a calculator.

∴ So, about 471 cubic centimeters of salsa are missing from the jar.

EXAMPLE 4 Real-Life Application

About how many gallons of water does the watercooler bottle contain? ($1 \text{ ft}^3 \approx 7.5$ gal)

(A) 5.3 gallons (B) 10 gallons (C) 17 gallons (D) 40 gallons

Find the volume of the cylinder. The diameter is 1 foot. So, the radius is 0.5 foot.

$V = Bh$ Write formula for volume.

$= \pi(0.5)^2(1.7)$ Substitute.

$= 0.425\pi \approx 1.3352$ Use a calculator.

So, the bottle contains about 1.3352 cubic feet of water. To find the number of gallons it contains, multiply by the conversion factor $\frac{7.5 \text{ gal}}{1 \text{ ft}^3}$.

$$1.3352 \text{ ft}^3 \times \frac{7.5 \text{ gal}}{1 \text{ ft}^3} \approx 10 \text{ gal}$$

∴ The watercooler bottle contains about 10 gallons of water. So, the correct answer is (B).

On Your Own

Now You're Ready
Exercise 12

3. **WHAT IF?** In Example 3, the height of the salsa in the jar is 5 centimeters. How much salsa is missing from the jar?
4. A cylindrical water tower has a diameter of 15 meters and a height of 5 meters. About how many gallons of water can the tower contain? ($1 \text{ m}^3 \approx 264$ gal)

8.1 Exercises

Check It Out
Help with Homework
BigIdeasMath.com

Vocabulary and Concept Check

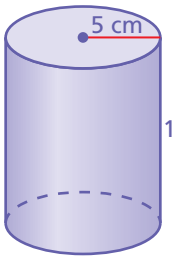
1. DIFFERENT WORDS, SAME QUESTION Which is different? Find "both" answers.

How much does it take to fill the cylinder?

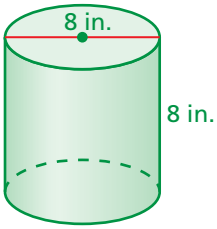
What is the capacity of the cylinder?

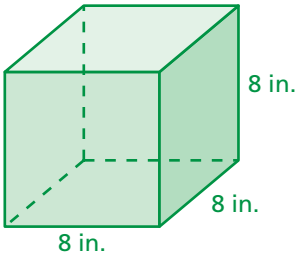
How much does it take to cover the cylinder?

How much does the cylinder contain?



2. REASONING Without calculating, which of the solids has the greater volume? Explain.



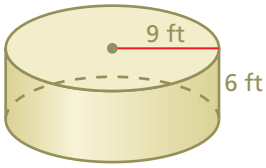


Practice and Problem Solving

Find the volume of the cylinder. Round your answer to the nearest tenth.

1

3.



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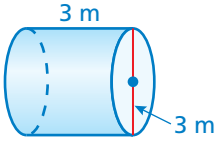

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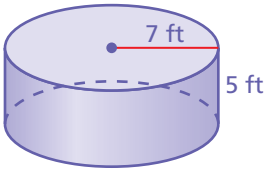

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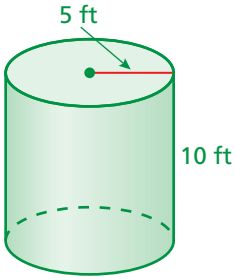

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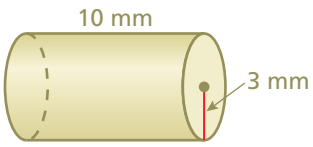

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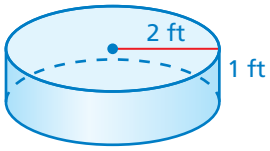

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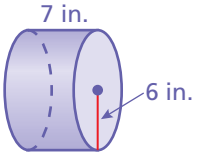
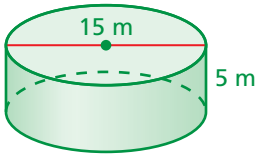
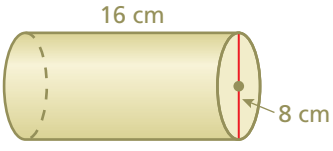


10.



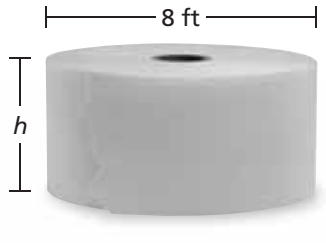
11.



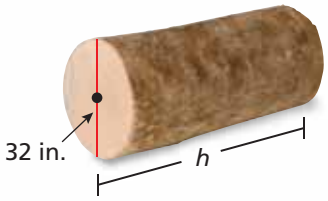
4 **12. SWIMMING POOL** A cylindrical swimming pool has a diameter of 16 feet and a height of 4 feet. About how many gallons of water can the pool contain? Round your answer to the nearest whole number. ($1 \text{ ft}^3 \approx 7.5$ gal)

Find the missing dimension of the cylinder. Round your answer to the nearest whole number.

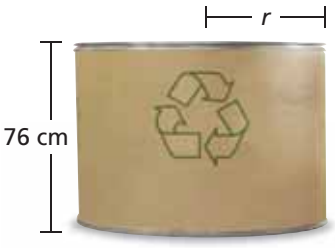
13. Volume $= 250 \text{ ft}^3$



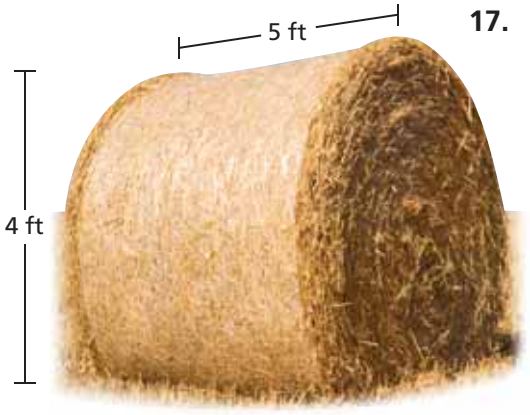
14. Volume $= 10{,}000\pi \text{ in.}^3$



15. Volume $= 600{,}000 \text{ cm}^3$



16. CRITICAL THINKING How does the volume of a cylinder change when its diameter is halved? Explain.



Round hay bale

17. MODELING A traditional "square" bale of hay is actually in the shape of a rectangular prism. Its dimensions are 2 feet by 2 feet by 4 feet. How many square bales contain the same amount of hay as one large "round" bale?

18. ROAD ROLLER A tank on a road roller is filled with water to make the roller heavy. The tank is a cylinder that has a height of 6 feet and a radius of 2 feet. One cubic foot of water weighs 62.5 pounds. Find the weight of the water in the tank.

19. VOLUME A cylinder has a surface area of 1850 square meters and a radius of 9 meters. Estimate the volume of the cylinder to the nearest whole number.

20. Problem Solving Water flows at 2 feet per second through a pipe with a diameter of 8 inches. A cylindrical tank with a diameter of 15 feet and a height of 6 feet collects the water.

a. What is the volume, in cubic inches, of water flowing out of the pipe every second?

b. What is the height, in inches, of the water in the tank after 5 minutes?

c. How many minutes will it take to fill 75% of the tank?

Fair Game Review What you learned in previous grades & lessons

Tell whether the triangle with the given side lengths is a right triangle. *(Section 7.5)*

21. 20 m, 21 m, 29 m **22.** 1 in., 2.4 in., 2.6 in. **23.** 5.6 ft, 8 ft, 10.6 ft

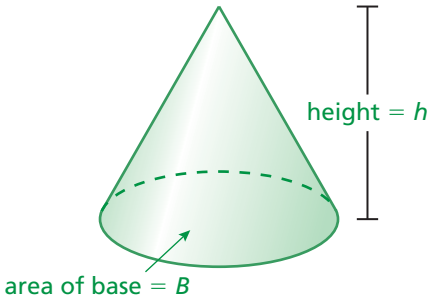
24. MULTIPLE CHOICE Which ordered pair is the solution of the linear system $3x + 4y = -10$ and $2x - 4y = 0$? *(Section 5.3)*

(A) $(-6, 2)$ (B) $(2, -6)$ (C) $(-2, -1)$ (D) $(-1, -2)$

8.2 Volumes of Cones

Essential Question How can you find the volume of a cone?

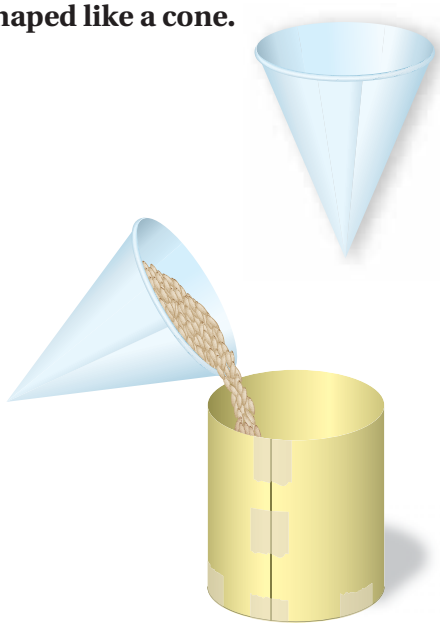
You already know how the volume of a pyramid relates to the volume of a prism. In this activity, you will discover how the volume of a cone relates to the volume of a cylinder.



1 ACTIVITY: Finding a Formula Experimentally

Work with a partner. Use a paper cup that is shaped like a cone.

- Estimate the height of the cup.
- Trace the top of the cup on a piece of paper. Find the diameter of the circle.
- Use these measurements to draw a net for a cylinder with the same base and height as the paper cup.
- Cut out the net. Then fold and tape it to form an open cylinder.
- Fill the paper cup with rice. Then pour the rice into the cylinder. Repeat this until the cylinder is full. How many cones does it take to fill the cylinder?
- Use your result to write a formula for the volume of a cone.



2 ACTIVITY: Summarizing Volume Formulas

Work with a partner. You can remember the volume formulas for prisms, cylinders, pyramids, and cones with just two concepts.

Volumes of Prisms and Cylinders

Volume = Area of base × ____

Volumes of Pyramids and Cones

Volume = ____ Volume of prism or cylinder with same base and height

Make a list of all the formulas you need to remember to find the area of a base. Talk about strategies for remembering these formulas.

COMMON CORE

Geometry

In this lesson, you will

- find the volumes of cones.
- find the heights of cones given the volumes.
- solve real-life problems.

Learning Standard 8.G.9

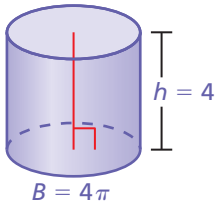
3 ACTIVITY: Volumes of Oblique Solids

Work with a partner. Think of a stack of paper. When you adjust the stack so that the sides are oblique (slanted), do you change the volume of the stack? If the volume of the stack does not change, then the formulas for volumes of right solids also apply to oblique solids.

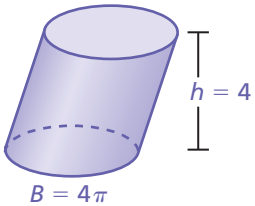
Math Practice 2

Use Equations

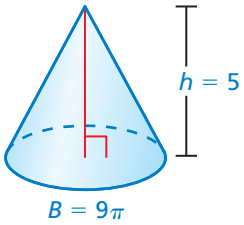
What equation would you use to find the volume of the oblique solid? Explain.



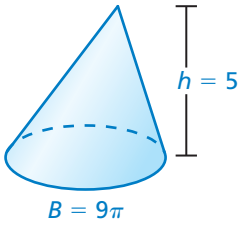
Right cylinder



Oblique cylinder



Right cone



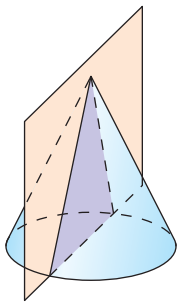
Oblique cone

What Is Your Answer?

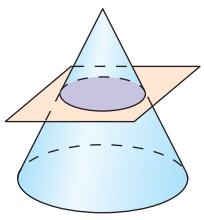
4. **IN YOUR OWN WORDS** How can you find the volume of a cone?

5. Describe the intersection of the plane and the cone. Then explain how to find the volume of each section of the solid.

a.



b.



Practice Use what you learned about the volumes of cones to complete Exercises 4–6 on page 344.

8.2 Lesson

Check It Out
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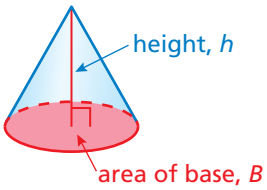
Key Idea

Volume of a Cone

Words The volume V of a cone is one-third the product of the area of the base and the height of the cone.



Algebra $V = \frac{1}{3}Bh$

Area of base

Height of cone

Study Tip

The *height* of a cone is the perpendicular distance from the base to the vertex.

EXAMPLE 1 Finding the Volume of a Cone

Find the volume of the cone. Round your answer to the nearest tenth.

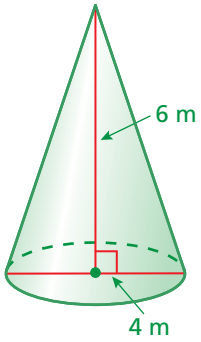
The diameter is 4 meters. So, the radius is 2 meters.

$V = \frac{1}{3}Bh$ Write formula for volume.

$= \frac{1}{3}\pi(2)^2(6)$ Substitute.

$= 8\pi \approx 25.1$ Use a calculator.

∴ The volume is about 25.1 cubic meters.



Study Tip

Because $B = \pi r^2$, you can use $V = \frac{1}{3}\pi r^2 h$ to find the volume of a cone.

EXAMPLE 2 Finding the Height of a Cone

Find the height of the cone. Round your answer to the nearest tenth.

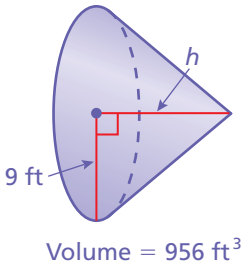
$V = \frac{1}{3}Bh$ Write formula for volume.

$956 = \frac{1}{3}\pi(9)^2(h)$ Substitute.

$956 = 27\pi h$ Simplify.

$11.3 \approx h$ Divide each side by 27π.

∴ The height is about 11.3 feet.

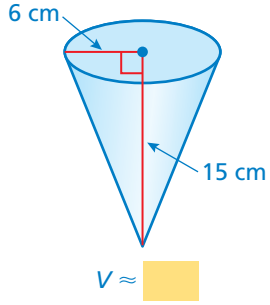


On Your Own

Now You're Ready
Exercises 4–12
and 15–17

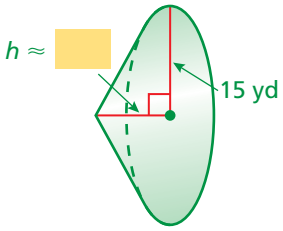
Find the volume V or height h of the cone. Round your answer to the nearest tenth.

1.



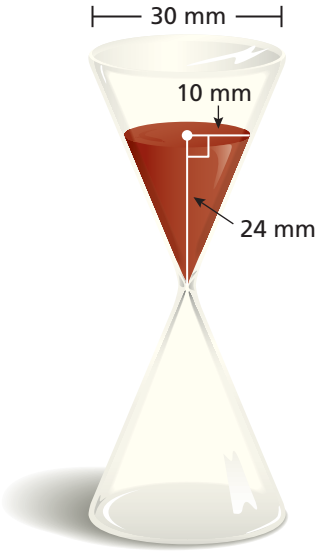
$V \approx$ ☐

2.



Volume = 7200 yd^3

EXAMPLE 3 Real-Life Application



You must answer a trivia question before the sand in the timer falls to the bottom. The sand falls at a rate of 50 cubic millimeters per second. How much time do you have to answer the question?

Use the formula for the volume of a cone to find the volume of the sand in the timer.

$$V = \frac{1}{3}Bh \qquad \text{Write formula for volume.}$$

$$= \frac{1}{3}\pi(10)^2(24) \qquad \text{Substitute.}$$

$$= 800\pi \approx 2513 \qquad \text{Use a calculator.}$$

The volume of the sand is about 2513 cubic millimeters. To find the amount of time you have to answer the question, multiply the volume by the rate at which the sand falls.

$$2513 \text{ mm}^3 \times \frac{1 \text{ sec}}{50 \text{ mm}^3} = 50.26 \text{ sec}$$

∴ So, you have about 50 seconds to answer the question.

On Your Own

3. **WHAT IF?** The sand falls at a rate of 60 cubic millimeters per second. How much time do you have to answer the question?

4. **WHAT IF?** The height of the sand in the timer is 12 millimeters, and the radius is 5 millimeters. How much time do you have to answer the question?

8.2 Exercises

Check It Out
Help with Homework
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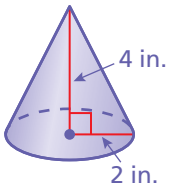
Vocabulary and Concept Check

1. **VOCABULARY** Describe the height of a cone.
2. **WRITING** Compare and contrast the formulas for the volume of a pyramid and the volume of a cone.
3. **REASONING** You know the volume of a cylinder. How can you find the volume of a cone with the same base and height?

Practice and Problem Solving

Find the volume of the cone. Round your answer to the nearest tenth.

1

4.


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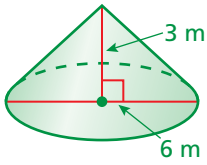

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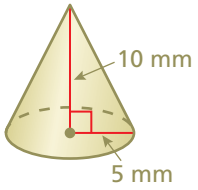

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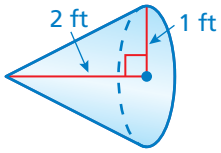

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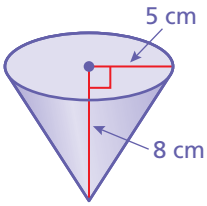

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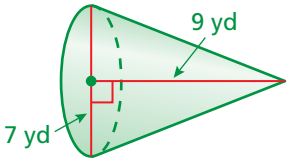

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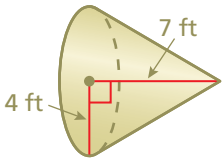

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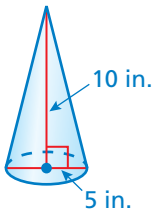

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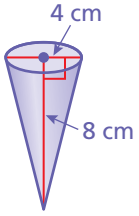
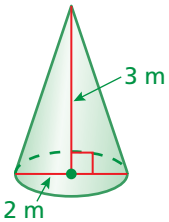
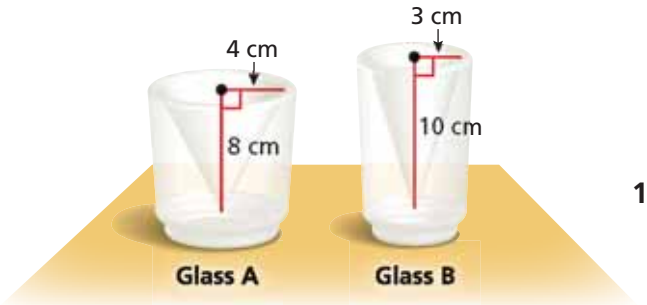


13. **ERROR ANALYSIS** Describe and correct the error in finding the volume of the cone.



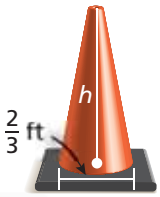
$$V = \frac{1}{3}Bh$$
$$= \frac{1}{3}(\pi)(2)^2(3)$$
$$= 4\pi \text{ m}^3$$



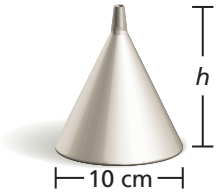
14. **GLASS** The inside of each glass is shaped like a cone. Which glass can hold more liquid? How much more?

Find the missing dimension of the cone. Round your answer to the nearest tenth.

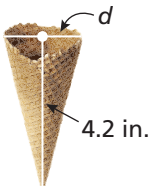
15. Volume $= \frac{1}{18}\pi$ ft^3

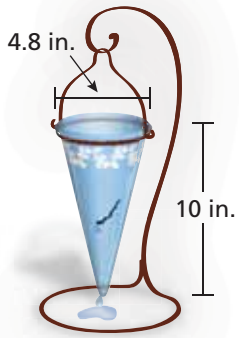


16. Volume $= 225$ cm^3



17. Volume $= 3.6$ in.^{3}





18. **REASONING** The volume of a cone is 20π cubic meters. What is the volume of a cylinder with the same base and height?

19. **VASE** Water leaks from a crack in a vase at a rate of 0.5 cubic inch per minute. How long does it take for 20% of the water to leak from a full vase?

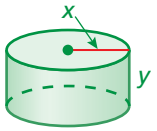
20. **LEMONADE STAND** You have 10 gallons of lemonade to sell. (1 gal ≈ 3785 cm^3)

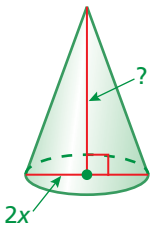
a. Each customer uses one paper cup. How many paper cups will you need?

b. The cups are sold in packages of 50. How many packages should you buy?

c. How many cups will be left over if you sell 80% of the lemonade?

21. **STRUCTURE** The cylinder and the cone have the same volume. What is the height of the cone?





22. **Critical Thinking** In Example 3, you use a different timer with the same dimensions. The sand in this timer has a height of 30 millimeters. How much time do you have to answer the question?

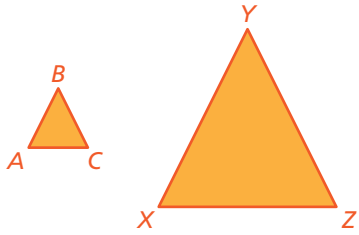
Fair Game Review What you learned in previous grades & lessons

The vertices of a figure are given. Rotate the figure as described. Find the coordinates of the image. *(Section 2.4)*

23. $A(-1, 1)$, $B(2, 3)$, $C(2, 1)$
90° counterclockwise about vertex A

24. $E(-4, 1)$, $F(-3, 3)$, $G(-2, 3)$, $H(-1, 1)$
180° about the origin

25. **MULTIPLE CHOICE** $\triangle ABC \sim \triangle XYZ$ by a scale factor of 3. How many times greater is the area of $\triangle XYZ$ than the area of $\triangle ABC$? *(Section 2.6)*



(A) $\frac{1}{9}$ (B) $\frac{1}{3}$

(C) 3 (D) 9

8.1–8.2 Quiz

Check It Out
Progress Check
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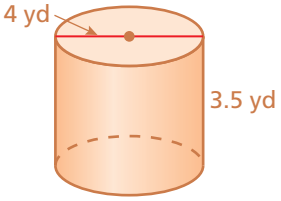
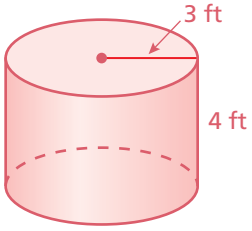
Find the volume of the solid. Round your answer to the nearest tenth. *(Section 8.1 and Section 8.2)*

1.



2.



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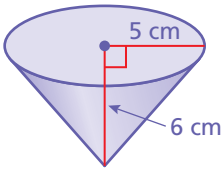

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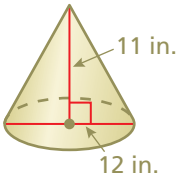
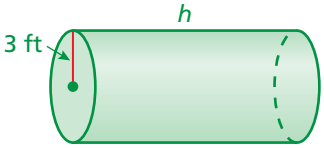


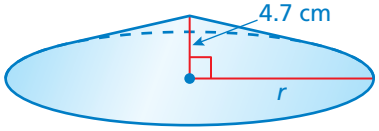
Find the missing dimension of the solid. Round your answer to the nearest tenth. *(Section 8.1 and Section 8.2)*

5.



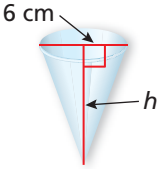
Volume = 340 ft^3

6.

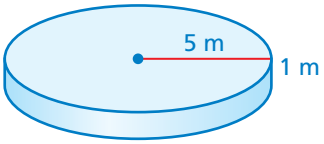


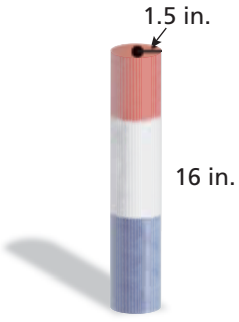
Volume = 938 cm^3

7. PAPER CONE The paper cone can hold 84.78 cubic centimeters of water. What is the height of the cone? *(Section 8.2)*



8. GEOMETRY Triple both dimensions of the cylinder. How many times greater is the volume of the new cylinder than the volume of the original cylinder? *(Section 8.1)*





9. SAND ART There are 42.39 cubic inches of blue sand and 28.26 cubic inches of red sand in the cylindrical container. How many cubic inches of white sand are in the container? *(Section 8.1)*

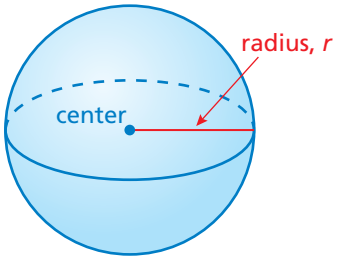
10. JUICE CAN You are buying two cylindrical cans of juice. Each can holds the same amount of juice. What is the height of Can B? *(Section 8.1)*

Can A Can B

8.3 Volumes of Spheres

Essential Question How can you find the volume of a sphere?

A **sphere** is the set of all points in space that are the same distance from a point called the *center*. The *radius r* is the distance from the center to any point on the sphere.

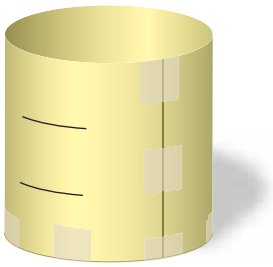


A sphere is different from the other solids you have studied so far because it does not have a base. To discover the volume of a sphere, you can use an activity similar to the one in the previous section.

1 ACTIVITY: Exploring the Volume of a Sphere

Work with a partner. Use a plastic ball similar to the one shown.

- Estimate the diameter and the radius of the ball.
- Use these measurements to draw a net for a cylinder with a diameter and a height equal to the diameter of the ball. How is the height h of the cylinder related to the radius r of the ball? Explain.



- Cut out the net. Then fold and tape it to form an open cylinder. Make two marks on the cylinder that divide it into thirds, as shown.
- Cover the ball with aluminum foil or tape. Leave one hole open. Fill the ball with rice. Then pour the rice into the cylinder. What fraction of the cylinder is filled with rice?

Common Core

Geometry

In this lesson, you will

- find the volumes of spheres.
- find the radii of spheres given the volumes.
- solve real-life problems.

Learning Standard 8.G.9

2 ACTIVITY: Deriving the Formula for the Volume of a Sphere

Work with a partner. Use the results from Activity 1 and the formula for the volume of a cylinder to complete the steps.

$V = \pi r^2 h$ Write formula for volume of a cylinder.

$= \frac{\square}{\square}\pi r^2 h$ Multiply by $\frac{\square}{\square}$ because the volume of a sphere is $\frac{\square}{\square}$ of the volume of the cylinder.

$= \frac{\square}{\square}\pi r^2 \square$ Substitute $\square$ for h.

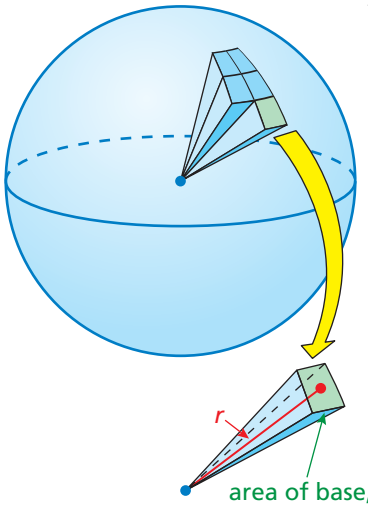
$= \frac{\square}{\square}\pi \square$ Simplify.

Math Practice 4

Analyze Relationships

What is the relationship between the volume of a sphere and the volume of a cylinder? How does this help you derive a formula for the volume of a sphere?

3 ACTIVITY: Deriving the Formula for the Volume of a Sphere



Work with a partner. Imagine filling the inside of a sphere with n small pyramids. The vertex of each pyramid is at the center of the sphere. The height of each pyramid is approximately equal to r, as shown. Complete the steps. (The surface area of a sphere is equal to $4\pi r^2$.)

$V = \frac{1}{3}Bh$ Write formula for volume of a pyramid.

$= n\frac{1}{3}B\square$ Multiply by the number of small pyramids n and substitute $\square$ for h.

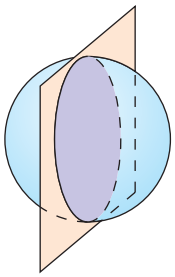
$= \frac{1}{3}(4\pi r^2)\square$ $4\pi r^2 \approx n \cdot \square$

Show how this result is equal to the result in Activity 2.

What Is Your Answer?

4. **IN YOUR OWN WORDS** How can you find the volume of a sphere?

5. Describe the intersection of the plane and the sphere. Then explain how to find the volume of each section of the solid.



Practice Use what you learned about the volumes of spheres to complete Exercises 3–5 on page 352.

8.3 Lesson

Check It Out
Lesson Tutorials
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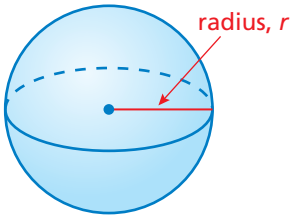
Key Vocabulary
sphere, *p. 348*
hemisphere, *p. 351*

Key Idea

Volume of a Sphere

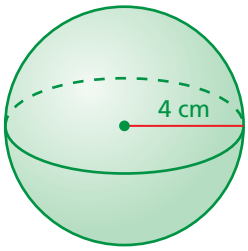
Words The volume V of a sphere is the product of $\frac{4}{3}\pi$ and the cube of the radius of the sphere.

Algebra $V = \frac{4}{3}\pi r^3$ (cube of radius of sphere)



EXAMPLE 1 Finding the Volume of a Sphere

Find the volume of the sphere. Round your answer to the nearest tenth.



$V = \frac{4}{3}\pi r^3$ Write formula for volume.

$= \frac{4}{3}\pi (4)^3$ Substitute 4 for r.

$= \frac{256}{3}\pi$ Simplify.

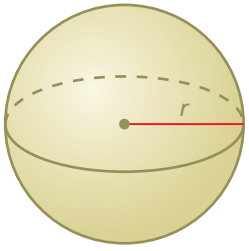
≈ 268.1 Use a calculator.

∴ The volume is about 268.1 cubic centimeters.

EXAMPLE 2 Finding the Radius of a Sphere

Find the radius of the sphere.

Volume = 288π in.^{3}



$V = \frac{4}{3}\pi r^3$ Write formula.

$288\pi = \frac{4}{3}\pi r^3$ Substitute.

$288\pi = \frac{4\pi}{3} r^3$ Multiply.

$\frac{3}{4\pi} \cdot 288\pi = \frac{3}{4\pi} \cdot \frac{4\pi}{3} r^3$ Multiplication Property of Equality

$216 = r^3$ Simplify.

$6 = r$ Take the cube root of each side.

∴ The radius is 6 inches.

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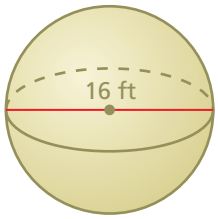
On Your Own

Now You're Ready
Exercises 3–11

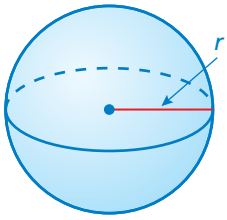
Find the volume V or radius r of the sphere. Round your answer to the nearest tenth, if necessary.

1.



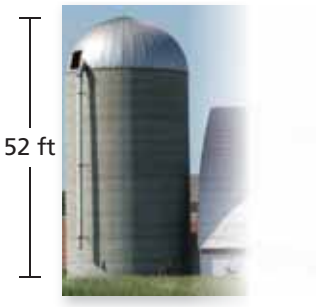
$V \approx$ ____

2.



Volume $= 36\pi$ m^3

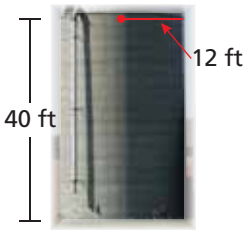
EXAMPLE 3 Finding the Volume of a Composite Solid



A hemisphere is one-half of a sphere. The top of the silo is a hemisphere with a radius of 12 feet. What is the volume of the silo? Round your answer to the nearest thousand.

The silo is made up of a cylinder and a hemisphere. Find the volume of each solid.

Cylinder

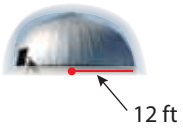


$V = Bh$

$= \pi(12)^2(40)$

$= 5760\pi$

Hemisphere



$V = \frac{1}{2} \cdot \frac{4}{3}\pi r^3$

$= \frac{1}{2} \cdot \frac{4}{3}\pi(12)^3$

$= 1152\pi$

Study Tip

In Example 3, the height of the cylindrical part of the silo is the difference of the silo height and the radius of the hemisphere.

$52 - 12 = 40$ ft

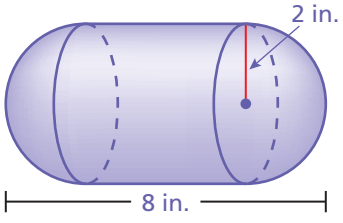
∴ So, the volume is $5760\pi + 1152\pi = 6912\pi \approx 22{,}000$ cubic feet.

On Your Own

Now You're Ready
Exercises 14–16

Find the volume of the composite solid. Round your answer to the nearest tenth.

3.



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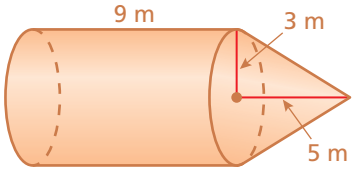

8.3 Exercises

Check It Out
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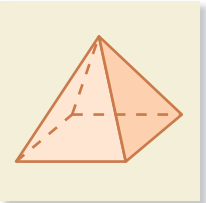
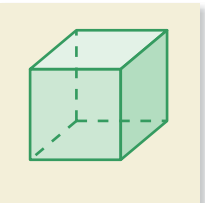
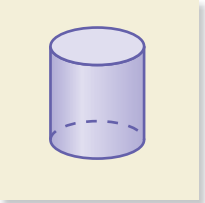
Vocabulary and Concept Check

1. **VOCABULARY** How is a sphere different from a hemisphere?
2. **WHICH ONE DOESN'T BELONG?** Which figure does *not* belong with the other three? Explain your reasoning.

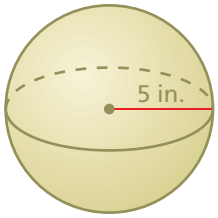





Practice and Problem Solving

Find the volume of the sphere. Round your answer to the nearest tenth.

1

3.


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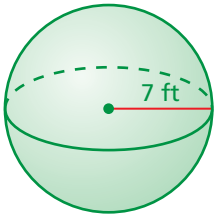

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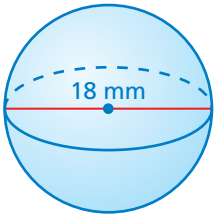

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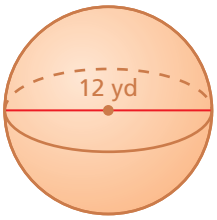

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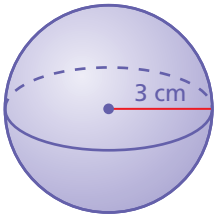

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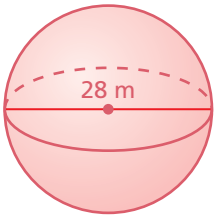


Find the radius of the sphere with the given volume.

2

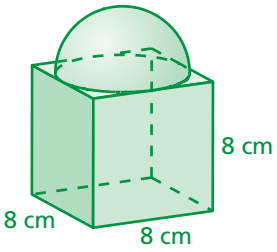
9. Volume $= 972\pi$ mm^3
10. Volume $= 4.5\pi$ cm^3
11. Volume $= 121.5\pi$ ft^3

12. **GLOBE** The globe of the Moon has a radius of 10 inches. Find the volume of the globe. Round your answer to the nearest whole number.

13. **SOFTBALL** A softball has a volume of $\frac{125}{6}\pi$ cubic inches. Find the radius of the softball.

Find the volume of the composite solid. Round your answer to the nearest tenth.

14.



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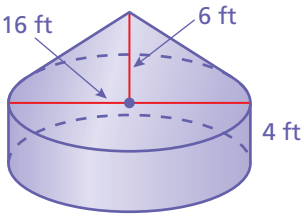

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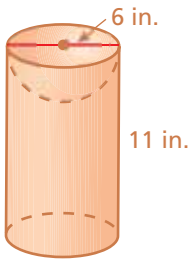


17. **REASONING** A sphere and a right cylinder have the same radius and volume. Find the radius r in terms of the height h of the cylinder.

18. **PACKAGING** A cylindrical container of three rubber balls has a height of 18 centimeters and a diameter of 6 centimeters. Each ball in the container has a radius of 3 centimeters. Find the amount of space in the container that is not occupied by rubber balls. Round your answer to the nearest whole number.

Volume $= 4500\pi$ in.^{3}

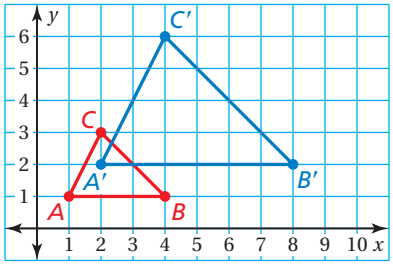
19. **BASKETBALL** The basketball shown is packaged in a box that is in the shape of a cube. The edge length of the box is equal to the diameter of the basketball. What is the surface area and the volume of the box?

20. **Logic** Your friend says that the volume of a sphere with radius r is four times the volume of a cone with radius r. When is this true? Justify your answer.

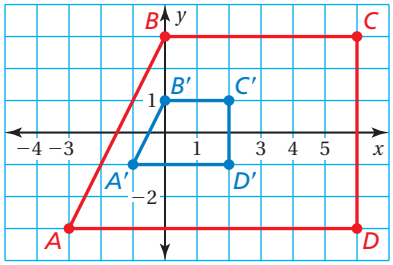
Fair Game Review What you learned in previous grades & lessons

The blue figure is a dilation of the red figure. Identify the type of dilation and find the scale factor. *(Section 2.7)*

21.



22.



23. **MULTIPLE CHOICE** A person who is 5 feet tall casts a 6-foot-long shadow. A nearby flagpole casts a 30-foot-long shadow. What is the height of the flagpole? *(Section 3.4)*

(A) 25 ft (B) 29 ft (C) 36 ft (D) 40 ft

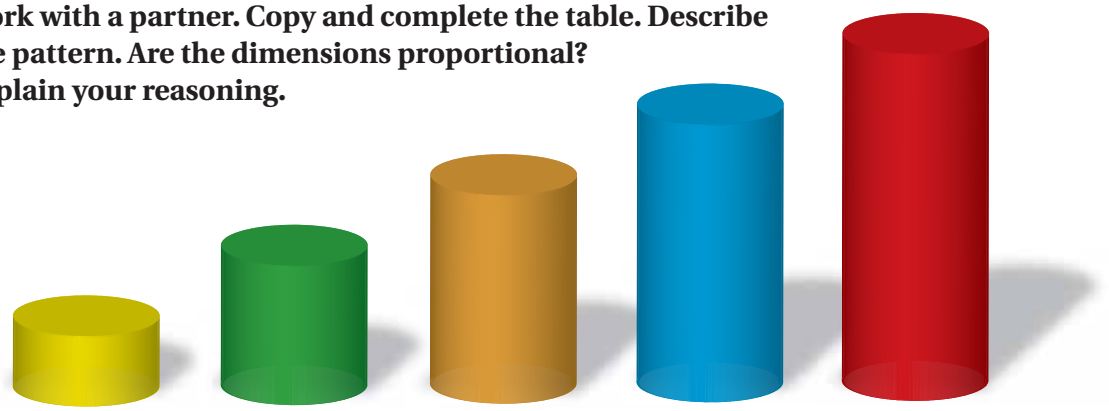
8.4 Surface Areas and Volumes of Similar Solids

Essential Question When the dimensions of a solid increase by a factor of k, how does the surface area change? How does the volume change?

1 ACTIVITY: Comparing Surface Areas and Volumes

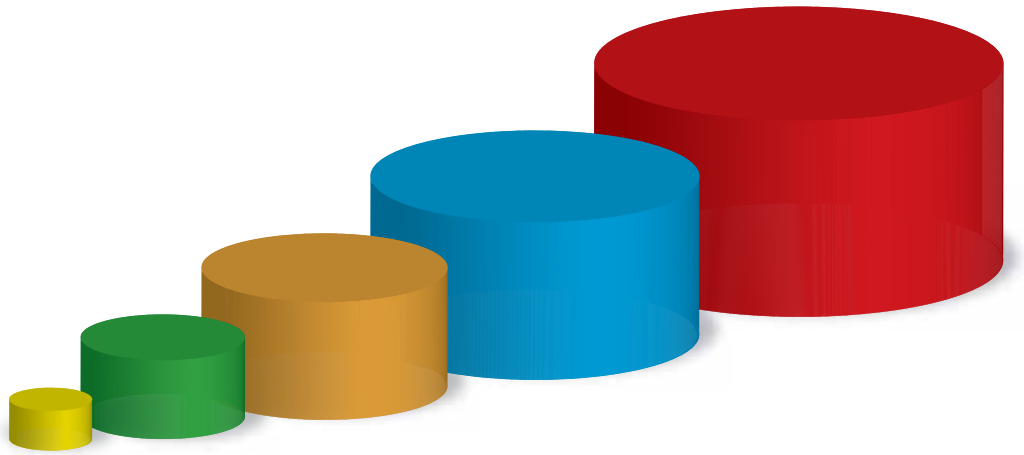
Work with a partner. Copy and complete the table. Describe the pattern. Are the dimensions proportional? Explain your reasoning.

a.



Radius	1	1	1	1	1
Height	1	2	3	4	5
Surface Area					
Volume					

b.



Radius	1	2	3	4	5
Height	1	2	3	4	5
Surface Area					
Volume					

Common Core

Geometry

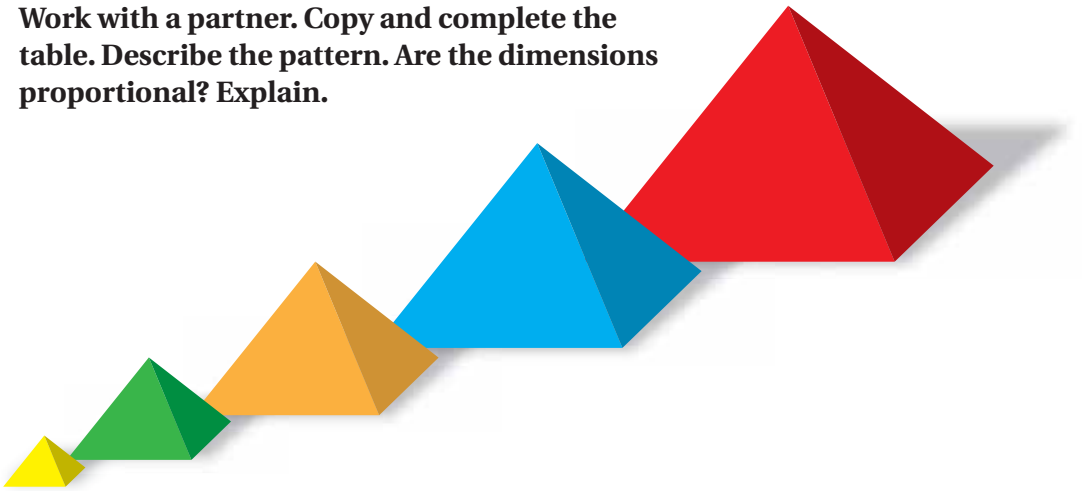
In this lesson, you will

- identify similar solids.
- use properties of similar solids to find missing measures.
- understand the relationship between surface areas of similar solids.
- understand the relationship between volumes of similar solids.
- solve real-life problems.

Applying Standard 8.G.9

2 ACTIVITY: Comparing Surface Areas and Volumes

Work with a partner. Copy and complete the table. Describe the pattern. Are the dimensions proportional? Explain.



Math Practice 8

Repeat Calculations

Which calculations are repeated? How does this help you describe the pattern?

Base Side	6	12	18	24	30
Height	4	8	12	16	20
Slant Height	5	10	15	20	25
Surface Area					
Volume					

What Is Your Answer?

3. **IN YOUR OWN WORDS** When the dimensions of a solid increase by a factor of k, how does the surface area change?

4. **IN YOUR OWN WORDS** When the dimensions of a solid increase by a factor of k, how does the volume change?

5. **REPEATED REASONING** All the dimensions of a prism increase by a factor of 5.

   a. How many times greater is the surface area? Explain.

   5 10 25 125

   b. How many times greater is the volume? Explain.

   5 10 25 125

Practice Use what you learned about surface areas and volumes of similar solids to complete Exercise 3 on page 359.

8.4 Lesson

Check It Out
Lesson Tutorials
BigIdeasMath.com

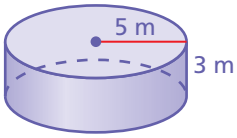
Key Vocabulary
similar solids, *p. 356*

Similar solids are solids that have the same shape and proportional corresponding dimensions.

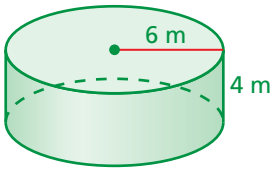
EXAMPLE 1 Identifying Similar Solids

Which cylinder is similar to Cylinder A?

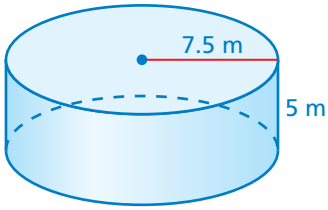
Cylinder B



Cylinder A



Cylinder C



Check to see if corresponding dimensions are proportional.

Cylinder A and Cylinder B

$\frac{\text{Height of A}}{\text{Height of B}} = \frac{4}{3}$ $\quad$ $\frac{\text{Radius of A}}{\text{Radius of B}} = \frac{6}{5}$

Not proportional

Cylinder A and Cylinder C

$\frac{\text{Height of A}}{\text{Height of C}} = \frac{4}{5}$ $\quad$ $\frac{\text{Radius of A}}{\text{Radius of C}} = \frac{6}{7.5} = \frac{4}{5}$

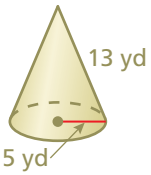
Proportional

∴ So, Cylinder C is similar to Cylinder A.

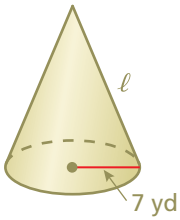
EXAMPLE 2 Finding Missing Measures in Similar Solids

The cones are similar. Find the missing slant height ℓ.

Cone X



Cone Y



$\frac{\text{Radius of X}}{\text{Radius of Y}} = \frac{\text{Slant height of X}}{\text{Slant height of Y}}$

$\frac{5}{7} = \frac{13}{\ell}$ Substitute.

$5\ell = 91$ Cross Products Property

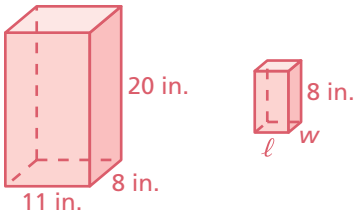
$\ell = 18.2$ Divide each side by 5.

∴ The slant height is 18.2 yards.

On Your Own

Now You're Ready
Exercises 4–9

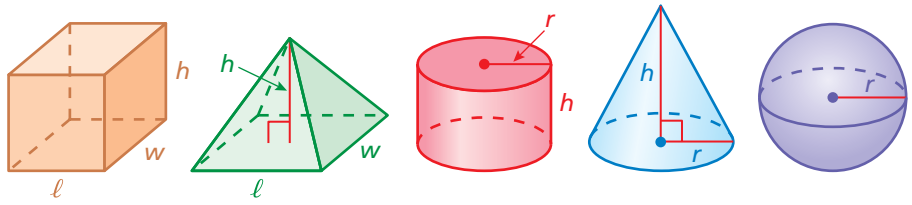
1. Cylinder D has a radius of 7.5 meters and a height of 4.5 meters. Which cylinder in Example 1 is similar to Cylinder D?
2. The prisms at the right are similar. Find the missing width and length.



Multi-Language Glossary at BigIdeasMath.com

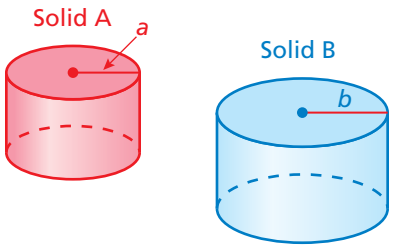
Key Ideas

Linear Measures



Surface Areas of Similar Solids

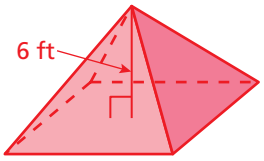
When two solids are similar, the ratio of their surface areas is equal to the square of the ratio of their corresponding linear measures.



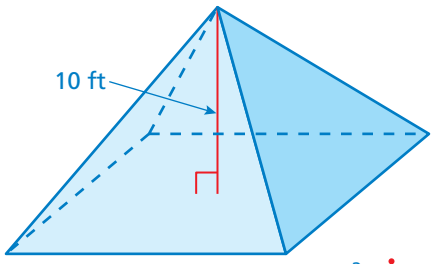
$$\frac{\text{Surface Area of A}}{\text{Surface Area of B}} = \left(\frac{a}{b}\right)^2$$

EXAMPLE 3 Finding Surface Area

Pyramid A



Pyramid B



Surface Area = 600 ft²

The pyramids are similar. What is the surface area of Pyramid A?

$$\frac{\text{Surface Area of A}}{\text{Surface Area of B}} = \left(\frac{\text{Height of A}}{\text{Height of B}}\right)^2$$

$$\frac{S}{600} = \left(\frac{6}{10}\right)^2 \quad \text{Substitute.}$$

$$\frac{S}{600} = \frac{36}{100} \quad \text{Evaluate.}$$

$$\frac{S}{600} \cdot 600 = \frac{36}{100} \cdot 600 \quad \text{Multiplication Property of Equality}$$

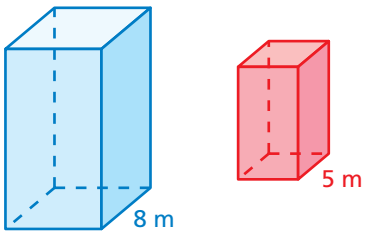
$$S = 216 \quad \text{Simplify.}$$

∴ The surface area of Pyramid A is 216 square feet.

On Your Own

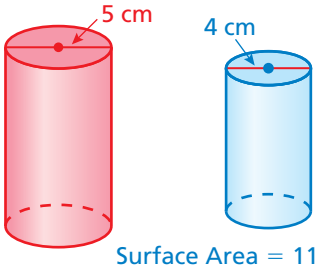
The solids are similar. Find the surface area of the red solid. Round your answer to the nearest tenth.

3.



Surface Area = 608 m²

4.



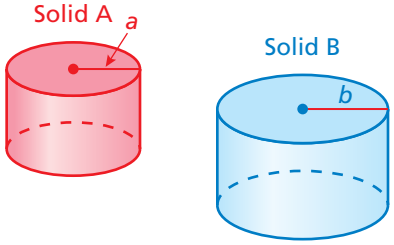
Surface Area = 110 cm²

Key Idea

Volumes of Similar Solids

When two solids are similar, the ratio of their volumes is equal to the cube of the ratio of their corresponding linear measures.

$$\frac{\text{Volume of A}}{\text{Volume of B}} = \left(\frac{a}{b}\right)^3$$



EXAMPLE 4 Finding Volume

Original Tank

Volume = 2000 ft^3

The dimensions of the touch tank at an aquarium are doubled. What is the volume of the new touch tank?

(A) 150 ft^3 (B) 4000 ft^3

(C) 8000 ft^3 (D) 16,000 ft^3

The dimensions are doubled, so the ratio of the dimensions of the original tank to the dimensions of the new tank is 1 : 2.

$$\frac{\text{Original volume}}{\text{New volume}} = \left(\frac{\text{Original dimension}}{\text{New dimension}}\right)^3$$

$$\frac{2000}{V} = \left(\frac{1}{2}\right)^3 \quad \text{Substitute.}$$

$$\frac{2000}{V} = \frac{1}{8} \quad \text{Evaluate.}$$

$$16{,}000 = V \quad \text{Cross Products Property}$$

∴ The volume of the new tank is 16,000 cubic feet. So, the correct answer is (D).

Study Tip

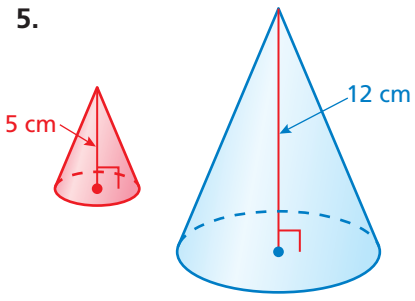
When the dimensions of a solid are multiplied by k, the surface area is multiplied by k^2 and the volume is multiplied by k^3.

On Your Own

Now You're Ready
Exercises 10–13

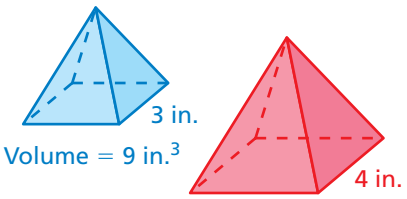
The solids are similar. Find the volume of the red solid. Round your answer to the nearest tenth.

5.



Volume = 288 cm^3

6.



Volume = 9 in.^{3}

8.4 Exercises

Check It Out
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Vocabulary and Concept Check

1. **VOCABULARY** What are similar solids?
2. **OPEN-ENDED** Draw two similar solids and label their corresponding linear measures.

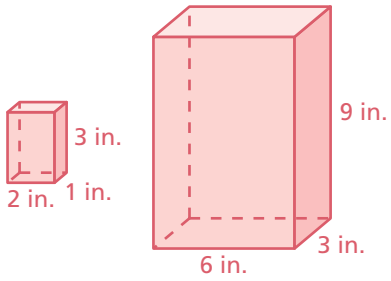
Practice and Problem Solving

3. **NUMBER SENSE** All the dimensions of a cube increase by a factor of $\frac{3}{2}$.
   a. How many times greater is the surface area? Explain.
   b. How many times greater is the volume? Explain.

Determine whether the solids are similar.

1

4.



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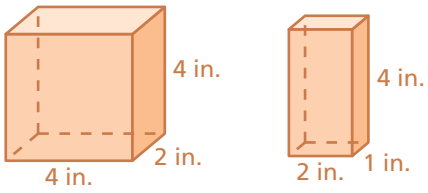

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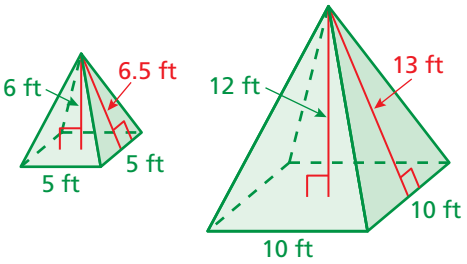

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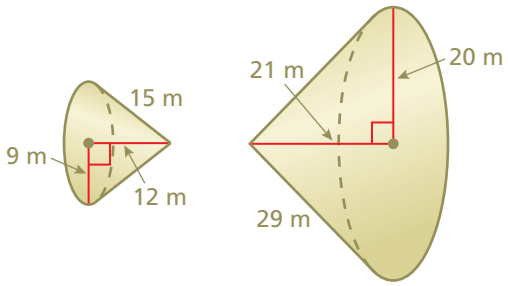
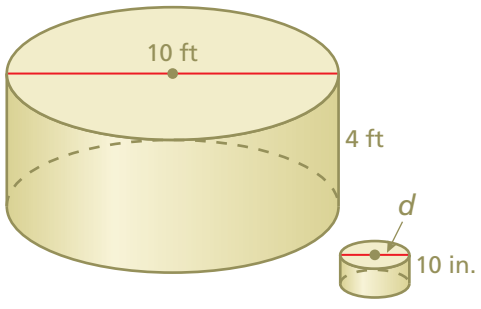


The solids are similar. Find the missing dimension(s).

2

8.



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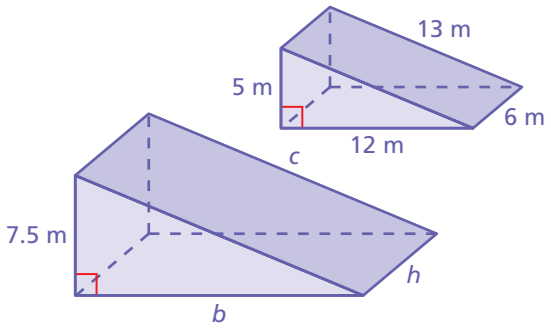
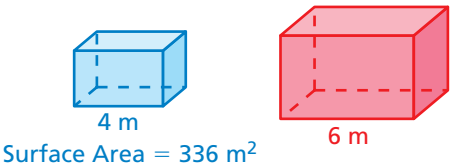


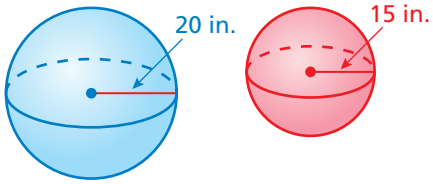
The solids are similar. Find the surface area S or volume V of the red solid. Round your answer to the nearest tenth.

3 4 **10.**



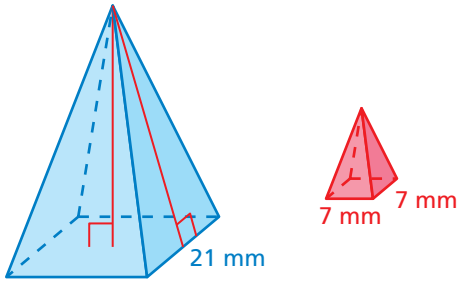
Surface Area = 336 m^2

11.



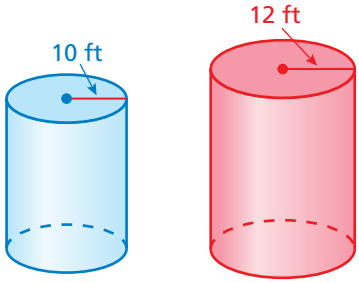
Surface Area = 1800 $in.^2$

12.



Volume = 5292 mm^3

13.



Volume = 7850 ft^3

14. ERROR ANALYSIS The ratio of the corresponding linear measures of two similar solids is 3 : 5. The volume of the smaller solid is 108 cubic inches. Describe and correct the error in finding the volume of the larger solid.

✗

$$\frac{108}{V} = \left(\frac{3}{5}\right)^2$$

$$\frac{108}{V} = \frac{9}{25}$$

$$300 = V$$

The volume of the larger solid is 300 cubic inches.

15. MIXED FRUIT The ratio of the corresponding linear measures of two similar cans of fruit is 4 to 7. The smaller can has a surface area of 220 square centimeters. Find the surface area of the larger can.

16. ENGINE The volume of a car engine is 390 cubic inches. Which scale model of the car has the greater engine volume, a 1 : 18 scale model or a 1 : 24 scale model? How much greater is it?

17. MARBLE STATUE You have a small marble statue of Wolfgang Mozart. It is 10 inches tall and weighs 16 pounds. The original statue is 7 feet tall.

a. Estimate the weight of the original statue. Explain your reasoning.

b. If the original statue were 20 feet tall, how much would it weigh?

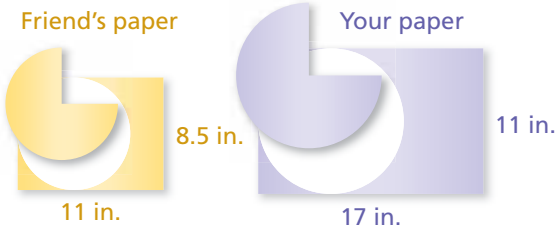
Wolfgang Mozart

18. REPEATED REASONING The largest doll is 7 inches tall. Each of the other dolls is 1 inch shorter than the next larger doll. Make a table that compares the surface areas and the volumes of the seven dolls.

19. Precision You and a friend make paper cones to collect beach glass. You cut out the largest possible three-fourths circle from each piece of paper.

a. Are the cones similar? Explain your reasoning.

b. Your friend says that because your sheet of paper is twice as large, your cone will hold exactly twice the volume of beach glass. Is this true? Explain your reasoning.



Fair Game Review What you learned in previous grades & lessons

Draw the figure and its reflection in the x-axis. Identify the coordinates of the image. *(Section 2.3)*

20. $A(1, 1), B(3, 4), C(4, 2)$

21. $J(-3, 0), K(-4, 3), L(-1, 4)$

22. MULTIPLE CHOICE Which system of linear equations has no solution? *(Section 5.4)*

(A) $y = 4x + 1$, $y = -4x + 1$

(B) $y = 2x - 7$, $y = 2x + 7$

(C) $3x + y = 1$, $6x + 2y = 2$

(D) $5x + y = 3$, $x + 5y = 15$

8.3–8.4 Quiz

Check It Out
Progress Check
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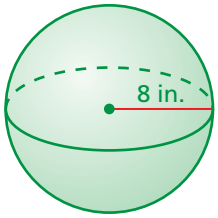
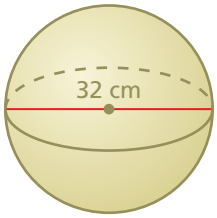
Find the volume of the sphere. Round your answer to the nearest tenth. *(Section 8.3)*

1.



2.

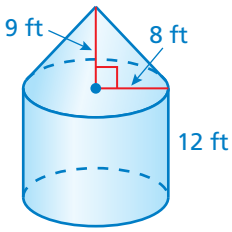


Find the radius of the sphere with the given volume. *(Section 8.3)*

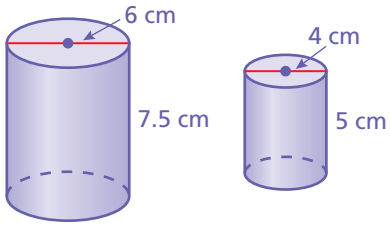
3. Volume $= 4500\pi$ yd^3

4. Volume $= \frac{32}{3}\pi$ ft^3

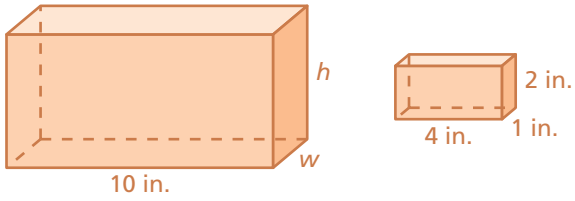
5. Find the volume of the composite solid. Round your answer to the nearest tenth. *(Section 8.3)*



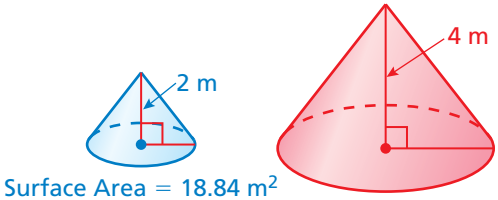
6. Determine whether the solids are similar. *(Section 8.4)*

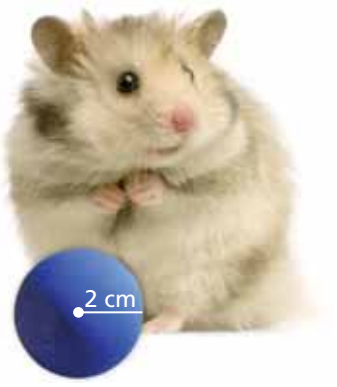


7. The prisms are similar. Find the missing width and height. *(Section 8.4)*



8. The solids are similar. Find the surface area of the red solid. *(Section 8.4)*

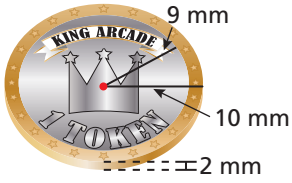




9. **HAMSTER** A hamster toy is in the shape of a sphere. What is the volume of the toy? Round your answer to the nearest whole number. *(Section 8.3)*

10. **JEWELRY BOXES** The ratio of the corresponding linear measures of two similar jewelry boxes is 2 to 3. The larger box has a volume of 162 cubic inches. Find the volume of the smaller jewelry box. *(Section 8.4)*

11. **ARCADE** You win a token after playing an arcade game. What is the volume of the gold ring? Round your answer to the nearest tenth. *(Section 8.3)*



8 Chapter Review

Check It Out
Vocabulary Help
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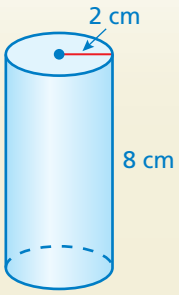
Review Key Vocabulary

sphere, *p. 348* hemisphere, *p. 351* similar solids, *p. 356*

Review Examples and Exercises

8.1 Volumes of Cylinders *(pp. 334–339)*

Find the volume of the cylinder. Round your answer to the nearest tenth.



$V = Bh$ Write formula for volume.

$= \pi(2)^2(8)$ Substitute.

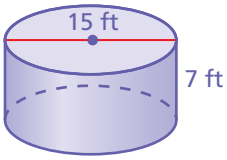
$= 32\pi \approx 100.5$ Use a calculator.

∴ The volume is about 100.5 cubic centimeters.

Exercises

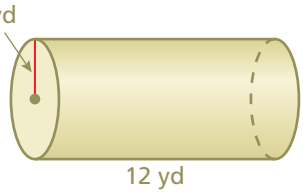
Find the volume of the cylinder. Round your answer to the nearest tenth.

1.

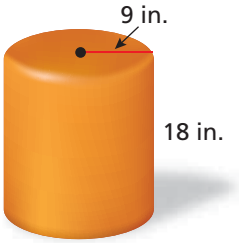


2.

3.

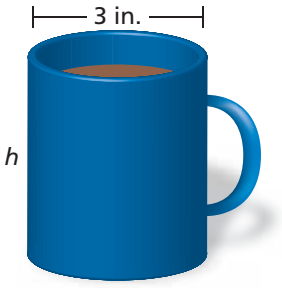


4.

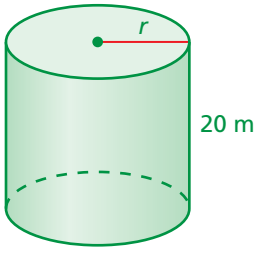


Find the missing dimension of the cylinder. Round your answer to the nearest whole number.

5. Volume $= 25$ in.^{3}



6. Volume $= 7599$ m^3



8.2 Volumes of Cones *(pp. 340–345)*

Find the height of the cone. Round your answer to the nearest tenth.

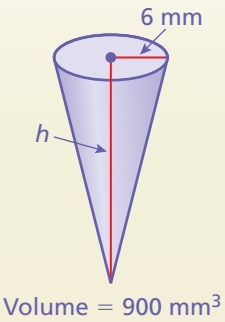
$V = \frac{1}{3}Bh$ Write formula for volume.

$900 = \frac{1}{3}\pi(6)^2(h)$ Substitute.

$900 = 12\pi h$ Simplify.

$23.9 \approx h$ Divide each side by 12π.

∴ The height is about 23.9 millimeters.

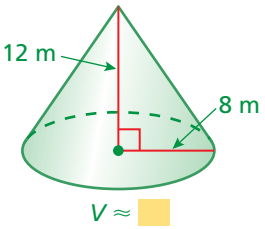


Volume = 900 mm^3

Exercises

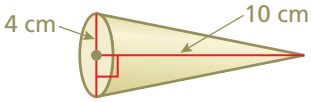
Find the volume V or height h of the cone. Round your answer to the nearest tenth.

7.



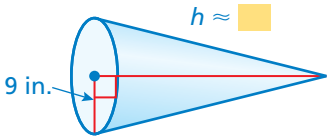
$V \approx$ ☐

8.



$V \approx$ ☐

9.



Volume = 3052 in.^{3}

8.3 Volumes of Spheres *(pp. 348–353)*

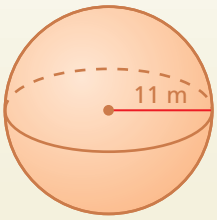
a. Find the volume of the sphere. Round your answer to the nearest tenth.

$V = \frac{4}{3}\pi r^3$ Write formula for volume.

$= \frac{4}{3}\pi(11)^3$ Substitute 11 for r.

$= \frac{5324}{3}\pi$ Simplify.

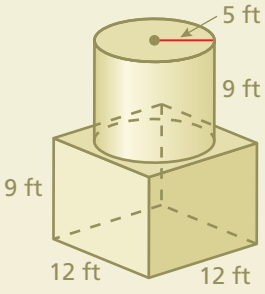
≈ 5575.3 Use a calculator.



∴ The volume is about 5575.3 cubic meters.

b. Find the volume of the composite solid. Round your answer to the nearest tenth.

Square Prism	*Cylinder*
$V = Bh$	$V = Bh$
$= (12)(12)(9)$	$= \pi(5)^2(9)$
$= 1296$	$= 225\pi \approx 706.9$

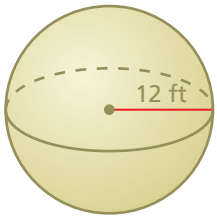


∴ So, the volume is about 1296 + 706.9 = 2002.9 cubic feet.

Exercises

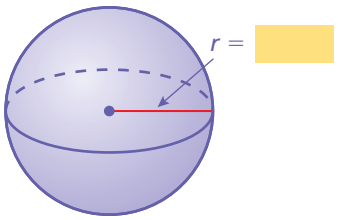
Find the volume V or radius r of the sphere. Round your answer to the nearest tenth, if necessary.

10.



$V \approx$ ____

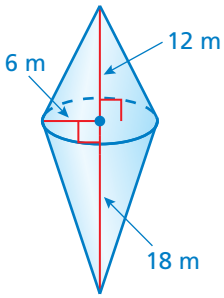
11.



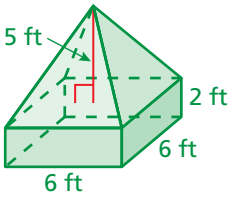
Volume = $12{,}348\pi$ in.^{3}

Find the volume of the composite solid. Round your answer to the nearest tenth.

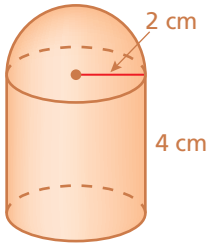
12.



13.



14.



8.4 Surface Areas and Volumes of Similar Solids *(pp. 354–361)*

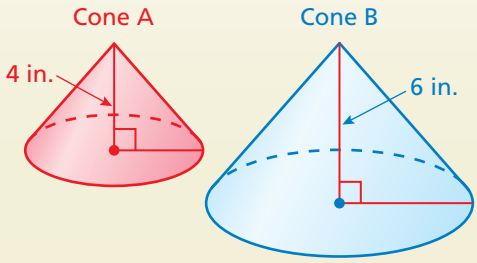
The cones are similar. What is the volume of the red cone? Round your answer to the nearest tenth.

$$\frac{\text{Volume of A}}{\text{Volume of B}} = \left(\frac{\text{Height of A}}{\text{Height of B}}\right)^3$$

$\frac{V}{157} = \left(\frac{4}{6}\right)^3$ Substitute.

$\frac{V}{157} = \frac{64}{216}$ Evaluate.

$V \approx 46.5$ Solve for V.



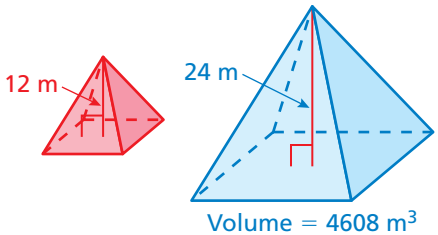
Volume = 157 in.^{3}

∴ The volume is about 46.5 cubic inches.

Exercises

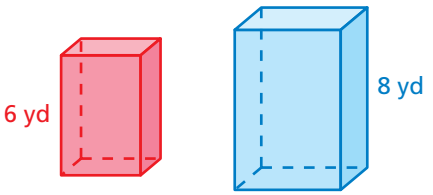
The solids are similar. Find the surface area S or volume V of the red solid. Round your answer to the nearest tenth.

15.



Volume = 4608 m^3

16.



Surface Area = 154 yd^2

8 Chapter Test

Check It Out
Test Practice
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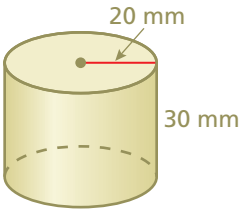
Find the volume of the solid. Round your answer to the nearest tenth.

1.



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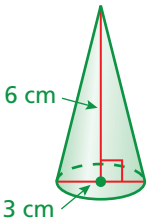

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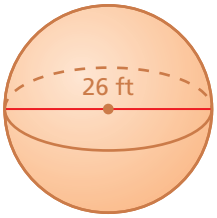

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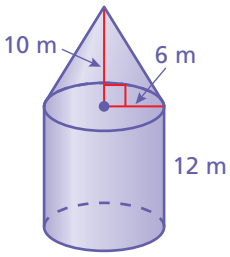
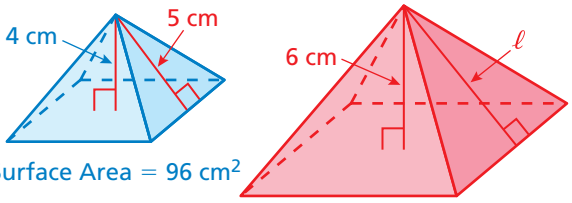
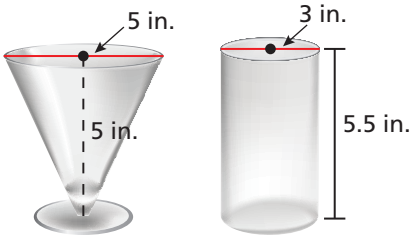


5. The pyramids are similar.

a. Find the missing dimension.

b. Find the surface area of the red pyramid.





6. SMOOTHIES You are making smoothies. You will use either the cone-shaped glass or the cylindrical glass. Which glass holds more? About how much more?

7. CONES The ratio of the corresponding linear measures of two similar cones is 3 to 4. The smaller cone has a volume of about 18 cubic inches. Find the volume of the larger cone. Round your answer to the nearest tenth.

8. OPEN-ENDED Draw two different composite solids that have the same volume but different surface areas. Explain your reasoning.

9. MILK Glass A has a diameter of 3.5 inches and a height of 4 inches. Glass B has a radius of 1.5 inches and a height of 5 inches. Which glass can hold more milk?

10. REASONING Without calculating, determine which solid has the greater volume. Explain your reasoning.